

## Effect of spin diffusion on spin torque in magnetic nanopillars

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We present systematic magnetoelectronic measurements of magnetic nanopillars with different structures of polarizing magnetic layers. The magnetic reversal at small magnetic field, the onset of magnetic dynamics at larger field, and the magnetoresistance exhibit a significant dependence on the type of the polarizing layer. We performed detailed quantitative modeling showing that the differences are caused by the effects of spin-dependent electron diffusion.

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Current-induced magnetic switching (CIMS) in nanoscale magnetic multilayers is caused by spin transfer (ST) from the conduction electrons to the magnetic layers, which occurs within atomic distances from the magnetic interfaces.<sup>1</sup> Nevertheless, electron diffusion in the layers can have a significant effect on ST.<sup>2-4</sup> For example, multiple electron scattering between two ferromagnets transfers angular momentum upon each reflection, while not significantly contributing to the net charge current  $I$ . Therefore, efficient utilization of electron scattering can reduce  $I$  required to manipulate magnetic devices. In a more subtle manifestation, spin-dependent electron diffusion causes an asymmetry between the ST in antiparallel (AP) and parallel (P) configurations of the magnetic layers,<sup>5</sup> in extreme cases leading to anomalous current-induced behaviors.<sup>6</sup>

Experimental studies of the effects of diffusion on ST (Refs. 6–10) have been hampered by the limited knowledge about the transport properties of individual layers and interfaces, which can significantly depend on the deposition techniques.<sup>11</sup> Since both the magnetoresistance (MR) and CIMS depend on the same spin-dependent transport properties, their simultaneous measurements and analysis within the same theoretical framework can lead to better understanding of the diffusion and its effect on ST. Here, we report on results for three multilayer structures elucidating different aspects of spin diffusion. Our calculations show that low-temperature results are consistent with the previously established transport properties of multilayers, while the temperature dependencies indicate that our present understanding of thermal spin-dependent scattering is inadequate.

Multilayers  $\text{Cu}(50)/\text{F}_1/\text{Cu}(10)/\text{F}_2/\text{Cu}(200)$  with identical free layers  $\text{F}_2=\text{Py}(5)$ ,  $\text{Py}=\text{Ni}_{80}\text{Fe}_{20}$ , and different polarizers  $\text{F}_1$  were deposited at room temperature (RT) by magnetron sputtering at a base pressure of  $5 \times 10^{-9}$  Torr, in 5 mTorr of purified Ar. Thicknesses are in nanometers.  $\text{F}_1$  was  $\text{Co}(20)$ ,  $\text{Co}(3)$ , and  $\text{Fe}_{50}\text{Mn}_{50}(1)/\text{Cu}(1)/\text{Co}(3)$  in the three sample types labeled Co20, Co3, and FeMnCo3, respectively. Samples Co20 and Co3 allowed us to analyze the effects of spin diffusion in Co and the bottom Cu lead, respectively. Samples FeMnCo3 were not significantly affected by spin diffusion in  $\text{F}_1$  due to the short spin diffusion length  $l_{\text{sf}}$  of FeMn.<sup>11</sup>  $\text{F}_2$  and part of the  $\text{Cu}(10)$  spacer were patterned into a nanopillar with approximate dimensions of  $130 \times 60 \text{ nm}^2$ , while  $\text{F}_1$  was left extended with dimensions of several micrometers. A 30-nm-thick undoped Si layer insulating the leads was subsequently sputtered without break-

ing the vacuum to protect the nanopillar from oxidation, followed by deposition of a thick top Cu lead. We measured  $dV/dI$  with four-probe and lock-in detection. Positive  $I$  flowed from  $\text{F}_1$  to  $\text{F}_2$ . Magnetic field  $H$  was in the film plane and along the nanopillar easy axis. At least three nanopillars of each type were tested.

Figures 1(a) and 1(b) show  $dV/dI$  vs  $I$  for a Co20 sample, acquired at RT and 5 K, respectively. The data at small  $H=50 \text{ Oe}$  are characterized by hysteretic jumps to the P state with resistance  $R_P$  at  $I^- < 0$  and to the AP state with resistance  $R_{\text{AP}}$  at  $I^+ > 0$ . Large peaks appearing at  $H=300/360 \text{ Oe}$  are caused by the reversible transitions between the AP state and a dynamical state with resistance above  $R_P$ . The onset of the magnetic dynamics at  $I=I_C$  appears as a sharp increase in  $dV/dI$  nearly independent of  $H$  [1 kOe data in Figs. 1(a) and 1(b)]. The equality  $I_C \approx I^+$  (dashed line) indicates that the reversal occurs when large-amplitude dynamics is excited by ST.<sup>12</sup> Figure 1(c) summarizes the  $T$  dependence of  $I^+$  and  $I^-$ . Both dramatically decrease with increasing  $T < 130 \text{ K}$ , above which they remain nearly constant. A similar behavior was seen in Co/Cu/Co nanopillars.<sup>8</sup> Magnetic hysteresis remained square between 5

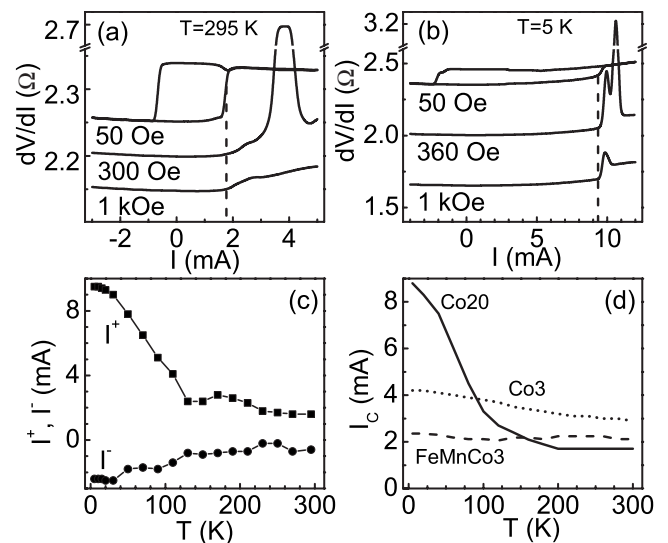


FIG. 1. (a)  $dV/dI$  vs  $I$  at labeled  $H$  and  $T=295 \text{ K}$ . Curves are offset for clarity. (b) Same as (a), at  $T=5 \text{ K}$ . (c)  $I^+$  and  $I^-$  vs  $T$  for a Co20 sample. (d)  $I_C$  vs  $T$  measured at  $H=500 \text{ Oe}$  for the three types of samples as labeled.

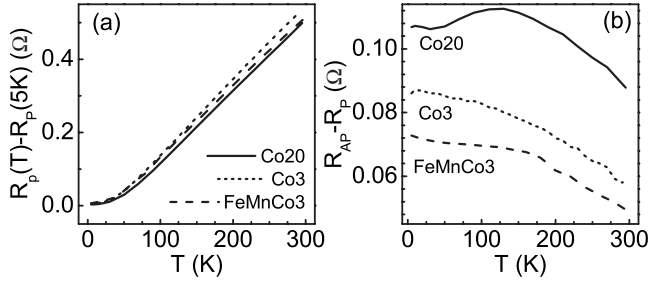


FIG. 2. (a) P-state resistances  $R_p$  offset by values at 5 K and (b) MR vs  $T$  for the three types of samples as labeled.

K and RT, eliminating magnetic inhomogeneity as the origin of these behaviors.

Some of the dependence on  $T$  in Fig. 1(c) may be caused by thermal activation. Therefore, we focus on the precession onset current  $I_C$ , which is a fundamental quantity predicted by the models, insensitive to thermal fluctuations and sample shape imperfections. It can be directly determined from the increase in  $dV/dI$  at  $H$  large enough to suppress hysteretic reversal. Additionally, inhomogeneity of  $F_1$  is minimized at large  $H$ . Figure 1(d) summarizes  $I_C$  vs  $T$  for all three different sample structures. FeMnCo3 data are approximately independent of  $T$ , while  $I_C$  for Co3 and Co20 decreases with  $T$ . Comparing panels (c) and (d) reveals that  $I_C$  for Co20 closely follows  $I^+$ . It is not possible to measure a similar excitation onset current  $I_C^-$  in the AP state, because the stability of the P state is not suppressed at any  $H$ . Below, we use  $I^-$  as an approximation for  $I_C^-$ .

Since  $F_2$  is identical in all samples, the different behaviors of  $I_C$  in Fig. 1(d) must be attributed to  $F_1$ . The difference between Co3 and FeMnCo3 is caused by the spin flipping in FeMn, which eliminates the spin diffusion in the bottom Cu(50) contact. The difference between the Co20 and Co3 data characterizes the effects of spin diffusion in Co vs Cu. We note that  $I_C$  is large in Co20 at low  $T$ , but it never diverges as would be expected if the sign of ST was reversed.<sup>6</sup>

Figures 2(a) and 2(b) show  $R_p(T) - R_p(0)$  and  $MR = R_{AP} - R_P$ . The values of  $R_p$  increased with  $T$  due to the increased magnon and phonon scattering, and were surprisingly consistent among the samples. Interestingly, there is a clear correlation between the variations in MR and  $I_C$ . A decrease in MR in Co20 with increasing  $T < 130$  coincides with a sharp decrease in  $I_C$ . A slow decrease in MR in Co20 with increasing  $T > 130$  K and in other samples at all  $T$  is correlated with small variations in  $I_C$ .

To analyze the current-induced behaviors, we performed simultaneous calculations of spin-dependent transport and ST based on the model proposed by Slonczewski<sup>5</sup> and supported by Boltzmann-equation calculations.<sup>13</sup> To calculate  $I_C$ , we first derive the expression for ST in terms of the spin-dependent transport properties for collinear magnetic configuration, which can be determined in the diffusive Valet-Fert approximation.<sup>14</sup> From Eq. (28) of Ref. 5, the spin-torque per unit area of the  $F_2/N$  interface is

$$L_R = \hbar[\Delta J_L - \Delta J_R \cos \theta]/2e \sin \theta, \quad (1)$$

where  $\Delta J_L$  ( $\Delta J_R$ ) is the difference between the spin-up and spin-down current densities in the Cu(10) spacer, in the ref-

erence frame determined by the magnetization of  $F_1$  ( $F_2$ ). Here,  $\hbar$  is the Planck constant,  $e$  is the electron charge, and  $\theta$  is the angle between the magnetizations of  $F_1$  and  $F_2$ . Expressing  $J_R$  through  $J_L$  and a similarly defined spin accumulation  $\Delta W_L$  via the spin-continuity equation (13) of Ref. 5, we obtain

$$L_R = \hbar \sin(\theta)(\Delta J_L - G\Delta W_L)/4e, \quad (2)$$

with  $G \approx 1 \text{ f } \Omega^{-1} \text{ m}^{-2}$  estimated by Slonczewski for Co/Cu interfaces. The usual expression for ST per unit area is  $\tau = \hbar j g(\theta) \sin \theta / e$ ,<sup>1</sup> where  $g(\theta)$  characterizes the angular dependence of ST and  $j$  is the current density. Comparing to Eq. (2), we obtain  $g(\theta) = (\Delta J_L - G\Delta W_L)/4j$ .  $I_C$  is determined by  $g(0)$ ,<sup>1</sup> which can be expressed in terms of the usual collinear transport parameters  $j_s \equiv \Delta J_L(\theta=0)$  and  $\Delta\mu \equiv \Delta W_L(\theta=0)$ ,<sup>14</sup>

$$I_C = 8\pi\alpha e M_2^2 V / (j_s/j - G\Delta\mu/j), \quad (3)$$

where  $\alpha \approx 0.03$  is the damping parameter<sup>15</sup> and  $V$  is the volume of  $F_2$ . We determined the magnetization  $M_2$  of Py by magnetometry of Py(5) films prepared under the same conditions as the nanopillars. It varied from 730 emu/cm<sup>3</sup> at 20 K to 675 emu/cm<sup>3</sup> at 300 K. The magnetization is lower than for bulk Py but consistent with the published data for films.<sup>16</sup> Extrapolation showed that  $M_2$  at 20 K was close to its residual value at 5 K.

Equation (3) expresses  $I_C$  in terms of  $\Delta\mu$  and  $j_s$ , the same quantities that determine MR in magnetic multilayers.  $I_C$  depends only on their ratios with  $j$ , which are independent of  $j$  in linear transport approximation. We calculated these ratios self-consistently using a one-dimensional diffusive Valet-Fert approximation employing the standard MR parameters: spin asymmetries  $\beta$ , renormalized resistivities  $\rho^* = \rho/(1-\beta^2)$ , spin diffusion lengths  $l_{sf}$  in the layers, and similarly defined parameters  $AR^*$ ,  $\gamma$ , and  $\delta$  for the interfaces.<sup>14</sup> We estimate these parameters from a combination of the published values<sup>11</sup> and our own measurements, as described below.

The resistivity  $\rho(T)$  of each layer in our samples provides essential information about electron diffusion. Because of variations among published values, we instead determined  $\rho$  from measurements of thin films prepared under the same conditions as the nanopillars, with thicknesses verified by x-ray reflectometry.  $\rho(T)$  was measured for 13-, 20-, and 40-nm-thick Py, Co, and Cu films at  $T=5-300$  K. The dependence of the residual resistivity on film thickness was consistent with the Fuchs-Sondheimer approximation, allowing us to extract the bulk residual values  $\rho_{Py}(0) = 11.3 \mu\Omega \text{ cm}$ ,  $\rho_{Co}(0) = 4.4 \mu\Omega \text{ cm}$ , and  $\rho_{Cu}(0) = 1.1 \mu\Omega \text{ cm}$ . The dependence of  $\rho$  on  $T$  for Co and Cu was consistent with the Bloch-Grüneisen approximation, while Py data exhibited a quadratic dependence. In all cases, the dependencies on  $T$  were consistent among the films with different thicknesses, allowing us to estimate  $\rho(T)$  for all the layers in the nanopillars.

To estimate  $l_{sf}(T)$ , we used its commonly assumed inverse relationship with  $\rho$ ,<sup>11</sup> along with the bulk residual values  $l_{sf,Py}(0) = 6 \text{ nm}$  and  $l_{sf,Cu}(0) = 300 \text{ nm}$  based on published measurements,<sup>11</sup> scaled by the somewhat different residual resistivities of our films. If scattering by thermal excitations

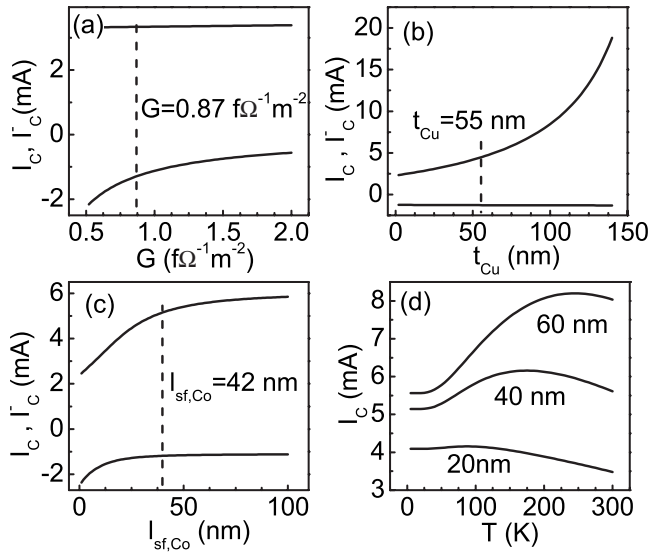


FIG. 3. (a) Calculated  $I_C$  and  $I_C^-$  vs  $G$  for FeMnCo3. (b) Same vs  $t_{Cu}$  for Co3. (c) Same vs  $l_{sf,Co}$  for Co20. (d)  $I_C$  vs  $T$  for Co20 samples, for the residual values of  $l_{sf,Co}$  as labeled.

does not flip electron spins, a weaker dependence  $l_{sf}(T) \propto \sqrt{1/\rho(T)}$  is possible. However, we show below that a dependence even stronger than  $1/\rho$  is more likely. We use  $\beta_{Py} = \gamma_{Py/Cu} = 0.7$ ,  $\gamma_{Co/Cu} = 0.8$ , and  $\beta_{Co} = 0.36$  for spin asymmetries,  $AR_{Co/Cu}^* = 0.55 f\Omega m^2$  and  $AR_{Py/Cu}^* = 0.5 f\Omega m^2$  for renormalized interface resistances, and  $\delta_{Co/Cu} = 0.2$  and  $\delta_{Py/Cu} = 0.25$  for spin flipping coefficients.<sup>8,11</sup> Their dependence on  $T$  is neglected due to the dominance of the band structure and impurity scattering far from the Curie point. For FeMn, we used  $l_{sf,FeMn} \approx 0.5$  nm, and  $\rho_{FeMn} = 87 \mu\Omega cm$ . Scattering at its interfaces was modeled by adding 0.5 nm to its nominal thickness. To account for the Cu contacts, the calculation included outer Cu layers of thickness  $t_{Cu}$  determined below. These layers were terminated with fictitious spin sinks.

To demonstrate that CIMS is extremely sensitive to the effects of diffusion, we now describe how our 5 K data can be fitted by appropriate choice of three parameters whose values have the largest uncertainty: conductance  $G$  in Eq. (2), effective MR-active thickness  $t_{Cu}$  of the Cu contacts, and spin diffusion length  $l_{sf,Co}$ . Calculations for FeMnCo3 were significantly affected only by  $G$ , which determines the asymmetry of CIMS. The values of  $I_C/I_C^-$  in milliamperes measured at 5 K for three FeMnCo3 samples were 2.3/0.8, 1.6/0.6, and 3.1/1.5, giving an average ratio  $I_C/I_C^- = 2.5 \pm 0.2$ . The calculated ratio increases from 1.46 at  $G = 0.5 f\Omega^{-1}m^{-2}$  to 6.1 at  $G = 2 f\Omega^{-1}m^{-2}$  [Fig. 3(a)]. The best approximation  $I_C/I_C^- = 3.34/1.27$  is obtained at  $G = 0.87 f\Omega^{-1}m^{-2}$ , in reasonable agreement with band-structure calculations.<sup>5,17</sup> We do not put uncertainty on this value because of the systematic errors introduced by the model and the uncertainties of other parameters.

Spin diffusion in the bottom Cu layer has little effect on Co20 and FeMnCo3 due to the spin relaxation in Co and FeMn, respectively. To determine  $t_{Cu}$ , we use the ratios  $I_C/I_C^-$  for three Co3 samples, 3.55/1.0, 4.6/1.5, and 4.2/1.2, giving an average ratio  $I_C/I_C^- = 3.4 \pm 0.1$ . The calculated

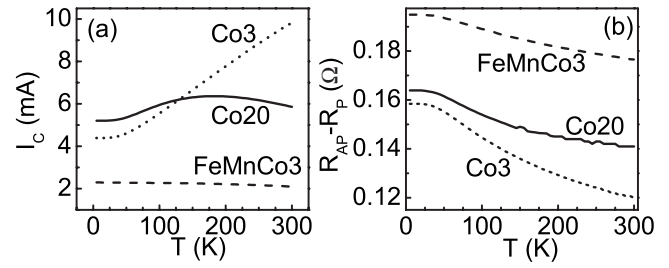


FIG. 4. (a) Calculated  $I_C$  vs  $T$  and (b) calculated MR vs  $T$  for three sample types as labeled.

$I_C/I_C^-$  increases from 1.9 for  $t_{Cu} = 0$  to 14 for  $t_{Cu} = 140$  nm [Fig. 3(b)] and eventually diverges at  $t_{Cu} = 200$  nm. The best agreement with data is obtained for  $t_{Cu} = 55$  nm, resulting in  $I_C/I_C^- = 4.4/1.3$ .

Lastly, diffusion in Co significantly affects CIMS in samples Co20, but not in Co3 and FeMnCo3. We determine  $l_{sf,Co}$  from the ratio  $I_C/I_C^-$  for five Co20 samples, 8.9/2.1, 7.3/1.6, 9.0/2.0, 8.5/2.0, and 8.0/1.7, giving an average ratio  $I_C/I_C^- = 4.5 \pm 0.1$ . Figure 3(c) illustrates that the calculated ratio  $I_C/I_C^-$  increases from 1.0 for  $l_{sf,Co} = 0$  to 5.2 for  $l_{sf,Co} = 100$  nm. The best agreement with the data is obtained for  $l_{sf,Co} = 42$  nm consistent with the published values.<sup>11</sup>

Despite the ability to model the 5 K data, the calculations did not reproduce the dramatic dependence of  $I_C$  on  $T$  in Fig. 1 (see below). Therefore, one can attempt to determine  $l_{sf,Co}$  from the dependence of  $I_C$  on  $T$ . Figure 3(d) shows calculations for the residual values  $l_{sf,Co} = 20, 40,$  and  $60$  nm. Large  $l_{sf,Co}$  results in  $I_C$  increasing with  $T$ , which is inconsistent with the data. Small  $l_{sf,Co}$  results in  $I_C$  decreasing with  $T$  in better qualitative agreement with data but gives unreasonably small  $I_C$  at 5 K. We thus return to the value determined from Fig. 3(c).

Figures 4(a) and 4(b) present the calculated  $I_C$  and MR vs  $T$  for the three sample types.  $I_C$  for FeMnCo3 did not significantly depend on  $T$ , in agreement with the data. This result supports our assumption that the temperature dependence of scattering at Co/Cu interfaces is negligible. However, the calculations for Co20 and Co3 fail to even qualitatively reproduce the data:  $I_C$  increases with increasing  $T < 180$  K for Co20, and at all  $T$  for Co3, in contrast to the measured decreases for both sample types.

To better understand the implications of the differences between the data and the calculations, we note that  $I_C$  in Fig. 3 increased whenever the effective MR-active resistance  $R_1$  of  $F_1$  increased. For a single-layer  $F_1$  with cross section  $A$ ,  $R_1 = l_{sf}\rho/A$ . For a multilayer  $F_1$ ,  $R_1$  is the sum of individual resistances weighed by the spin relaxation. For Co3 in Fig. 3(b),  $R_1$  increased with increasing thickness of the Cu contact. For Co20 in Fig. 3(c),  $R_1$  increased with increasing  $l_{sf,Co}$  due to the contribution from the outer Co/Cu interface and the bottom Cu contact. A similar relationship between  $I_C$  and  $R_1$  was established analytically.<sup>5,18</sup>

For Co3, the measured decrease in  $I_C$  with  $T$  indicates that  $R_1$  decreases with  $T$ , implying that  $t_{Cu}$  must decrease. We originally assumed that, because of the large  $l_{sf,Cu}$ , this parameter is determined by the current spreading in the leads, which is a purely geometrical factor for a homogeneous ma-

terial. Data for Co3 thus suggest that thermal electron scattering in the Cu contact may change the effective geometry of the contact.

For Co20, the situation is more complicated, because  $R_1$  is determined by a competition between the contribution of Co(20) increasing with  $T$  for  $l_{sf,Co} > 20$  nm, and the contributions of the bottom Cu(50) contact and the outer Co/Cu interface, which decrease with  $T$  due to stronger spin flipping in Co(20). In calculations, the former dominates at  $T < 180$  K, while the latter dominates at higher  $T$ , resulting in nonmonotonic  $I_C(T)$ . Both of these contributions to  $R_1$  reduce the polarizing properties of  $F_1$ , lowering MR [Fig. 4(b)]. In contrast, the measured increase in MR and dramatic decrease in  $I_C$  with increasing  $T < 130$  K suggest a significant reduction in  $R_1$ , and enhancement of the polarizing properties of  $F_1$ . To identify the origin of these behaviors, additional studies with different thicknesses of the polarizing Co layer may be warranted. Little is presently known about the temperature dependence of  $l_{sf,Co}(T)$ . We speculate that  $l_{sf,Co}$  may decrease with  $T$  faster than  $1/\rho_{Co}$ , resulting in reduction in the  $l_{sf,Co}\rho_{Co}$  contribution to  $R_1$ . The mechanism for such a rapid reduction in  $l_{sf}$  is presently unclear.

Our data may also indicate more fundamental limitations of the model. First, diffusive transport approximation may

not adequately describe transport across heterostructures consisting of nanometer-scale layers. Second, the transport properties of our heterostructures may be modified by the nonequilibrium electron distribution during CIMS. Finally, the  $s$ -like and  $d$ -like components of the electron wave may have different contributions to the transport properties and CIMS. Nonequilibrium distribution calculations involving realistic band structures may be needed to address the significance of these effects.<sup>19</sup>

To summarize, we performed magnetoelectronic measurements of nanopillars with three different polarizing magnetic layers. The samples exhibited different current-induced behaviors attributed to the spin diffusion in the polarizing layer. The calculations reproduced the low-temperature behaviors with reasonable values of transport parameters. However, temperature dependencies of magnetoresistance and current-induced switching indicate that the effects of thermal scattering on spin transport are not yet well understood.

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